

# Fermi-bounce Cosmology and scale invariant power-spectrum

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(Dated: February 25, 2014)*

We develop a novel non-singular bouncing cosmology, due to the non-trivial coupling of general relativity to fermionic fields. The resolution of the singularity arises from the negative energy density provided by fermions. Our theory is ghost-free because the fermionic operator that generates the bounce is equivalent to torsion, which has no kinetic terms. The physical system is minimal in that it consists of standard general relativity plus a topological sector for gravity, a U(1) gauge field reducing to radiation at late times and fermionic matter described by Dirac fields with a non-minimal coupling. We show that a scale invariant power-spectrum generated in the contracting phase can be recovered for a suitable choice of the fermion number density and the bare mass, hence providing a possible alternative to the inflationary scenario.

PACS numbers: 98.80.Cq, 98.80.Es, 04.60.Pp, 04.62.+v

It is well known that at initial times the Friedmann–Lemaître–Robertson–Walker (FLRW) metric of the Standard Big Bang cosmology suffers from singularities in all curvature invariants, as remarked in the pioneering work of Hawking and Penrose [1]. This theorem states that the initial singularity is unavoidable if space-time is described by General Relativity and if matter obeys the null energy condition. Over the years non-singular bouncing cosmologies have been proposed to avoid the big-bang singularity by obviating one or all of the assumptions behind the Hawking–Penrose theorem. However, a successful theory of the early universe must predict the observed nearly scale invariant spectrum of adiabatic fluctuations in the CMBR. Scale invariance was attempted in the context of bouncing models with a contracting phase such as Ekpyrotic [2], String Gas [3] and Pre-Big Bang scenarios [4]. On the other hand, it has proven difficult to obtain adiabatic scale invariant fluctuations in the contracting phase in a number of these models; mainly due to issues in resolving the singularity or mode matching between contracting and expanding phases [4].

In pioneering work by Brandenberger, Finelli and independently, Wands [4, 5] it was shown it is possible to generate a scale invariant power spectrum in a matter dominated contracting universe. These authors demonstrated a “duality” between the scale invariant power spectrum generated in the inflationary epoch and a contracting matter dominated phase. During the contracting phase, gauge invariant perturbations that cross the Hubble-scale are scale invariant if the scale factor grows as  $a(t) \sim (-t)^{2/3}$ . Furthermore, if the bounce is non-singular the scale-invariant modes can be matched to scale-invariant modes in the expanding phase.

A handful of matter bounce scenarios have since now been proposed [26], all of them based on fundamental scalar fields [2, 8–11]. In this letter we present a matter bounce scenario based on Dirac fermions, and specifi-

cally on the four-fermion interaction. In this model, general relativity is naturally extended to have topological terms which encode gravitational interactions with Dirac fermions [27]. These interactions naturally lead to Four-Fermion current densities, which modify the Friedman equations to have a negative energy density that redshifts like  $\sim a(t)^6$ . We show that the resulting bounce is non-singular provided that anisotropic stress is subdominant [28]. Moreover, we show for the first time that the adiabatic quantum fluctuation of the fermions in the contracting phase is indeed scale-invariant. Using the result of Brandenberger and Finelli, since the bounce is non-singular our scale-invariant curvature perturbation induced by the fermion quantum fluctuations will enter the expanding phase as a scale invariant fluctuation.

In what follows we provide our theoretical framework and conventions. For this purpose, we follow Refs. [6, 15–18]. We start by considering a generalization of the Einstein–Hilbert action that includes a topological sector of gravity and still provides the Einsteins’s equations: this is the Holst action for gravity, and cast in the Palatini formalism allows us to couple gravity to chiral fermions. We then couple this theory to a Dirac field  $\psi$ , whose complex conjugate reads  $\bar{\psi} = (\psi^*)^T \gamma^0$ . The action for the fermionic field is cast in terms of the Dirac matrices,  $\gamma^I$  with  $I = 0, \dots, 3$  and  $\gamma^5$ , expressed in the Dirac–Pauli basis. The action for pure gravity can be cast in terms of the gravitational field  $g_{\mu\nu} = e_\mu^I e_\nu^J \eta_{IJ}$ , where  $e_\mu^I$  is the tetrad/frame field (with inverse  $e_\mu^I$  and determinant  $e$ ), and the Lorentz connection  $\omega_\mu^{IJ}$  (whose curvature is  $F_{\mu\nu}^{IJ} = 2\partial_{[\mu}\omega_{\nu]}^{IJ} + [\omega_\mu, \omega_\nu]^{IJ}$ ). The action for the fermion fields involves the spinors  $\psi$  and  $\bar{\psi} = \psi^\dagger \gamma^0$ .

The total action is the sum of the non-minimal Einstein–Cartan–Holst (ECH) action plus the covariant Dirac action. The ECH action is (see [14]),

$$S_{\text{Holst}} = \frac{1}{2\kappa} \int_M d^4x |e| e_I^\mu e_J^\nu P^{IJ}{}_{KL} F_{\mu\nu}{}^{KL}(\omega), \quad (1)$$

where  $\kappa = 8\pi G_N$  is the reduced Planck length square and the operator  $P^{IJ}{}_{KL} = \delta_K^{[I} \delta_L^{J]} - \epsilon^{IJ}{}_{KL}/(2\gamma)$ ,  $\epsilon_{IJKL}$  being the Levi-Civita symbol, is defined in terms of the Barbero–Immirzi parameter  $\gamma$ , and can be inverted for  $\gamma^2 \neq -1$ . The Dirac action is  $S_{\text{Dirac}} = \frac{1}{2} \int d^4x |e| \mathcal{L}_{\text{Dirac}}$ , where

$$\mathcal{L}_{\text{Dirac}} = \frac{1}{2} \left[ \bar{\psi} \gamma^I e_I^\mu \left( 1 - \frac{i}{\alpha} \gamma_5 \right) i \nabla_\mu \psi - m \bar{\psi} \psi \right] + \text{h.c.}, \quad (2)$$

in which  $\alpha \in \mathbb{R}$  is the crucial non-minimal coupling parameter. The Einstein-Cartan action can be found if we consider  $S_{\text{ECH}} = S_{\text{GR}} + S_{\text{Dirac}}$  and  $\alpha = \gamma$ , with a term that reduces to the Nieh–Yan invariant [19] when the second Cartan structure equation holds. From the point of view of the Holst action (1), minimal coupling is recovered in the limit  $\alpha \rightarrow \pm\infty$ . Constraints on  $\alpha$  and  $\gamma$  can be derived from the four fermion axial-current Lagrangian (7), based on measurements of lepton-quark contact interactions [20, 21], but these are not stringent at all.

The covariant derivative for Dirac spinors is defined to be  $\nabla_\mu \equiv \partial_\mu + \frac{1}{4} \omega_\mu^{IJ} \gamma_{[I} \gamma_{J]}$ , while the field-strength of the Lorentz connection is obtained from  $[\nabla_\mu, \nabla_\nu] = \frac{1}{4} F_{\mu\nu}^{IJ} \gamma_{[I} \gamma_{J]}$ . Because of the presence of fermions, a torsional part of the connection enters the non-minimal ECH action. Nevertheless, the latter can be integrated out of the theory through the Cartan equation, which is found by varying the total action with respect to the connection  $\omega_\mu^{IJ}$ . We provide the usual definition of the contortion tensor, denoted as  $C_\mu^{IJ}$  and defined by  $(\nabla_\mu - \tilde{\nabla}_\mu) V_I = C_\mu{}^J{}_I V_J$ , where  $\tilde{\nabla}_\mu$  is the covariant derivative compatible with the tetrad  $e_\mu^I$  and  $V_J$  a vector in the internal space. The Cartan equation then relates the contortion tensor  $C_\mu^{IJ}$  to the fermionic currents and tetrad:

$$e_I^\mu C_{\mu JK} = \frac{\kappa}{4} \frac{\gamma}{\gamma^2 + 1} (\beta \epsilon_{IJKL} J^L - 2\theta \eta_{I[J} J_{K]}) , \quad (3)$$

$$J^L = \bar{\psi} \gamma^L \gamma_5 \psi ,$$

where the coefficients are functions of the free parameters within the non-minimal ECH theory,  $\beta = \gamma + 1/\alpha$  and  $\theta = 1 - \gamma/\alpha$ . Thanks to (3) the non-minimal ECH action can be completely recast in terms of the metric compatible connection, as a sum of the Einstein–Hilbert action and the Dirac action. The latter is now written in terms of metric compatible variables, and is further provided with a novel interaction term, which captures the new physics within the non-minimal ECH theory  $S_{\text{ECH}}$ . The theory then recasts as

$$S_{\text{ECH}} = S_{\text{GR}} + S_{\text{Dirac}} + S_{\text{Int}}, \quad (4)$$

where the Einstein–Hilbert action is expressed in terms of the mixed-indices Riemann tensor  $R_{\mu\nu}^{IJ} = F_{\mu\nu}^{IJ}[\tilde{\omega}(e)]$

$$S_{\text{GR}} = \frac{1}{2\kappa} \int_M d^4x |e| e_I^\mu e_J^\nu R_{\mu\nu}^{IJ}, \quad (5)$$

the Dirac action  $S_{\text{Dirac}}$  on curved space-time reads

$$S_{\text{Dirac}} = \frac{1}{2} \int_M d^4x |e| \left( \bar{\psi} \gamma^I e_I^\mu i \tilde{\nabla}_\mu \psi - m \bar{\psi} \psi \right) + \text{h.c.}, \quad (6)$$

and finally the interacting part of the theory is:

$$S_{\text{Int}} = -\xi \kappa \int_M d^4x |e| J^L J^M \eta_{LM}, \quad (7)$$

having defined the coefficient  $\xi$  as a function of the fundamental parameters of the theory

$$\xi := \frac{3}{16\gamma^2 + 1} \left( 1 + \frac{2}{\alpha\gamma} - \frac{1}{\alpha^2} \right). \quad (8)$$

In the early Universe, the overall physical picture may be captured if including gauge fields interactions. We then couple fermionic fields to U(1) gauge field (which can be eventually a dark-sector), and show in what follows that even U(1) fields' contribution to early Universe dynamics can be neglected. The total theory then reads

$$S_{\text{phys}} = S_{\text{ECH}} + S_{\text{U}(1)} + \tilde{S}_{\text{Int}}, \quad (9)$$

having introduced the U(1) action. The metric-compatible action for the U(1) gauge sector is made of the pure gauge part, namely

$$S_{\text{U}(1)} = \frac{1}{4} \int_M d^4x |e| F_{\mu\nu}(A) F_{\rho\sigma}(A) e_I^\rho e^{I\mu} e_J^\sigma e^{J\nu}, \quad (10)$$

where  $F_{\mu\nu}(A) = \partial_{[\mu} A_{\nu]}$  is the field-strength of the U(1) connection  $A_\mu$ , and of the interacting part

$$\tilde{S}_{\text{Int}} = q \int_M d^4x |e| A_\mu e_L^\mu \bar{\psi} \gamma^L \psi, \quad (11)$$

$q$  denoting the U(1) charge. The Einstein equations  $G_{\mu\nu} = \kappa T_{\mu\nu}$  provide the dynamics for the gravitational field  $e_\mu^I$ , and must be coupled to the equations of motion for fermionic matter and radiation. We have denoted with  $G_{\mu\nu}$  the Einstein tensor and

$$T_{\mu\nu} = \frac{e_\mu{}^I}{|e|} \frac{\delta}{\delta e_I^\nu} (|e| \mathcal{L}_{\text{matt}}) \quad (12)$$

the energy-momentum tensor sourced by fermionic matter and radiation in  $S_{\text{matt}} = \int_M d^4x |e| \mathcal{L}_{\text{matt}}$ , where  $S_{\text{matt}} := S_{\text{phys}} - S_{\text{GR}}$ . Concerning fermionic matter, the action reads  $S_{\text{fer}} := S_{\text{Dirac}} + S_{\text{Int}}$ , with

$$\mathcal{L}_{\text{fer}} = \frac{1}{2} \left( \bar{\psi} \gamma^I e_I^\mu \tilde{\nabla}_\mu \psi - m \bar{\psi} \psi \right) + \text{h.c.} - \xi \kappa J^L J_L, \quad (13)$$

which yields the energy-momentum tensor

$$T_{\mu\nu}^{\text{fer}} = \frac{1}{4} \bar{\psi} \gamma_I e_{(\mu}^I i \tilde{\nabla}_{\nu)} \psi + \text{h.c.} - g_{\mu\nu} \mathcal{L}_{\text{fer}}. \quad (14)$$

The energy-momentum tensor for the U(1) field is:

$$T_{\mu\nu}^{\text{U}(1)} = F_{\alpha\mu} F_{\nu}^{\alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} - e g_{\mu\nu} A_{\rho} \mathcal{J}^{\rho} - e A_{(\mu} \mathcal{J}_{\nu)}, \quad (15)$$

in which we have denoted the fermionic current as

$$\mathcal{J}^{\rho} = e_I^{\rho} \bar{\psi} \gamma^I \psi. \quad (16)$$

The Dirac equations on curved background for the interacting system are easily derived from  $\mathcal{L}_{\text{fer}}$ , yielding

$$\gamma^I e_I^{\mu} i \tilde{\nabla}_{\mu} \psi - m \psi = 2\xi \kappa (\bar{\psi} \psi + \bar{\psi} \gamma_5 \psi \gamma_5 + \bar{\psi} \gamma_I \psi \gamma^I) \psi, \quad (17)$$

in which we have used the Pauli-Fierz identity

$$\begin{aligned} (\bar{\psi} \gamma_5 \gamma^I \psi) (\bar{\psi} \gamma_5 \gamma_I \psi) &= \\ &= (\bar{\psi} \psi)^2 + (\bar{\psi} \gamma_5 \psi)^2 + (\bar{\psi} \gamma^I \psi) (\bar{\psi} \gamma_I \psi). \end{aligned}$$

*Non-Singular Bounce* — In previous bouncing models, the issue of the robustness of the singularity avoidance depended on whether quantum corrections (*i.e.* curvature or matter) were under-control at the bounce [4]. The advantage of our model is that the torsion in this scheme, which is responsible for the bounce, has no-kinetic term (an auxiliary field) and will not experience any quantum corrections as we approach the bounce.

We focus on the early Universe dynamics and assume symmetry reduction of  $e_{\mu}^I$  to Friedmann-Lemaître-Robertson-Walker (FLRW) type metrics, which in the comoving gauge reads  $e_0^I = \delta_0^I$  and  $e_j^I = \delta_j^I a(t)$ ,  $t$  denoting time. Homogeneity and isotropy on spatial hypersurfaces set up spatial components of the fermionic current to vanish and U(1) vector fields, when they are small enough not to spoil isotropy, to be  $A_{\mu} = (0, \tilde{A}(t))$ . The only non-trivial Maxwell equations are those ones involving the contravariant metric-independent components of the electric field  $E^i = \dot{A}^i$ , that is,  $\dot{E}^i + 3H E^i = 0$ , where  $H = \dot{a}/a$  is the Hubble parameter. Thus the electric field  $E^i$  redshifts as  $E^i \sim 1/a^3$ . The scaling for  $E_i$  provides the usual expression for the energy density of radiation  $\rho_{\text{rad}} \sim 1/a^4$  [29]. For a normalized scale factor, isotropy demands that for  $|E| = |\tilde{E}|/a^3$ , in which we use  $|\tilde{E}|^2 = \delta_{ab} E^a E^b$  and denote  $\tilde{E}_i$  as constants, both  $|\tilde{E}|^2 \ll m \bar{\psi} \psi$  and  $|\tilde{E}|^2 \ll \kappa J^L J_L$  hold.

As for the Dirac field components, the vanishing of the spatial fermionic current yields [12]  $\psi = (\psi_0, 0, 0, 0)$ . In the comoving gauge, the only non-vanishing spin connection components for  $\omega^{IJK} = \omega_{\mu}^{IJ} e_K^{\mu}$  are  $\omega_{0ij} = -\omega_{i0j} = -H \delta_{ij}$ . This implies  $\tilde{\nabla}_0 = \partial_0$  and  $\tilde{\nabla}_i = \partial_i + aH/2 \delta_{ij} \text{diag}(\sigma^j, -\sigma^j)$ , where  $\sigma^j$  denotes Pauli matrices. The Dirac equation then follows

$$\dot{\psi}_0 + \frac{3}{2} H \psi_0 + i(m + 4\xi \kappa \psi_0^* \psi_0) \psi_0 = 0, \quad (18)$$

in which  $*$  denotes complex conjugation. It is easy to derive the equation of motion for the bilinear  $\psi_0^* \psi_0$ ,

$$\frac{d}{dt} \psi_0^* \psi_0 + 3H \psi_0^* \psi_0 = 0, \quad (19)$$

which yields the familiar expression in terms of a constant initial density  $n_0$

$$\psi_0^* \psi_0 \sim \frac{n_0}{a^3}. \quad (20)$$

From the Einstein equations and the energy-momentum tensor expressions (14) and (15) we derive the Friedmann equations, which we cast in terms of the *ansatz* on  $\psi$  and then of the result for  $\rho_{\text{fer}}/m = \psi_0^* \psi_0$  in (20).  $G_0^0 = \kappa T_0^0$  recasts as

$$H^2 = \xi \frac{\kappa^2}{3} \frac{n_0^2}{a^6} + \frac{\kappa |\tilde{E}|^2}{6 a(t)^4} + \frac{m \kappa n_0}{3 a^3}. \quad (21)$$

Equation (21) entails an important term that is proportional to  $n_0^2$  and redshifts as  $a^6$ , the four-fermion interaction term, while the other terms are the electro-magnetic radiation, redshifting as  $a^4$ , and the matter energy density, redshifting as  $a^3$ . We now consider a contracting scale factor and immediately recognize that the bounce is due to the vanishing of the total energy density. As we approach the would be singularity (the scale-factor approaching zero), the negative energy four-fermion term dominates and drives the Hubble parameter to zero, resulting in a non-singular bounce.

Subtracting twice (21) to  $G = \kappa T$ , we derive

$$\dot{H} - H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6} \left( m \kappa \frac{n_0}{a^3} + \frac{\kappa |\tilde{E}|^2}{a(t)^4} + 4 \xi \kappa^2 \frac{n_0^2}{a^6} \right), \quad (22)$$

which is the second Friedmann equation. It is quite straightforward to bound the amplitude of the electric field to be consistent with the bounce, which is  $\kappa |\tilde{E}|^2 \ll m \kappa n_0$  and  $\kappa |\tilde{E}|^2 \ll \kappa^2 \rho_0^2$  hold. At the bounce,  $t = t_0$ , for  $|\tilde{E}| = 0$  the vanishing of  $H = H_0$  in (21) implies that the scale factor approaches a constant,  $a_0 = (-\xi \kappa n_0/m)^{1/3}$ . For negative values of  $\xi$  the scale factor  $a_0$  reaches its minimum, as from (22) one finds that  $\dot{H}_0 = -m^2/(3\xi)$ . Notice that both the bilinear  $\bar{\psi} \psi$  and the field  $\psi$  reach their maxima at  $t_0$ : although the effective potential in  $S_{\text{fer}}$  is unbounded in  $\psi$  when  $\xi$  negative, the gravitational bounce prevents from having a classically unbounded energy spectrum. It is then straightforward to find the deterministic evolution of the scale factor that leads to the bounce:

$$a = \left( \frac{3m\kappa n_0}{4} (t - t_0)^2 - \xi \frac{\kappa n_0}{m} \right)^{\frac{1}{3}}. \quad (23)$$

*Cosmological Curvature Perturbations* — Following [12], we introduce a quantity that is conserved on large

scales and can be related to CMBR temperature fluctuations, analogous to the Bardeen variable [30], *i.e.*

$$\zeta = \frac{\delta\rho}{\rho + p}. \quad (24)$$

After a little algebra and resorting to the Pauli-Fierz identity, we obtain:

$$\zeta = \frac{1}{m\bar{\psi}\psi + 2\xi\kappa J_L J_L} \left\{ (m + \xi\kappa\bar{\psi}\psi) (\delta\bar{\psi}\psi + \bar{\psi}\delta\psi) + \xi\kappa [\bar{\psi}\gamma_5\psi(\delta\bar{\psi}\gamma_5\psi + \bar{\psi}\gamma_5\delta\psi) + \bar{\psi}\gamma^L\psi(\delta\bar{\psi}\gamma_L\psi + \bar{\psi}\gamma_L\delta\psi)] \right\}.$$

On the back-ground solution,  $\zeta$  is further simplified to be

$$\zeta = f(t)(\delta\bar{\psi}\psi + \bar{\psi}\delta\psi), \quad \text{with } f(t) \simeq \frac{(1 - \xi\kappa\bar{\psi}\psi/m)}{\bar{\psi}\psi}. \quad (25)$$

The quantum fluctuations of the spinor field can be expanded as follows,

$$\delta\psi = \sum_h \int \frac{d^3k}{(2\pi)^{3/2}} \times (u_h(t, \vec{k})a(\vec{k}, h)e^{i\vec{k}\cdot\vec{x}} + v_h(t, \vec{k})b^\dagger(\vec{k}, h)e^{-i\vec{k}\cdot\vec{x}}). \quad (26)$$

The power spectrum  $\mathcal{P}(k)$  is implicitly defined in terms of the equal-time two-point function of  $\zeta$ :

$$\langle \zeta(t, \vec{x})\zeta(t, \vec{x} + \vec{r}) \rangle = \int \frac{dk}{k} \frac{\sin kr}{kr} \mathcal{P}(k), \quad (27)$$

where  $k=|\vec{k}|$  and so forth  $r=|\vec{r}|$ , and the expectation value is taken in the vacuum state defined by  $a|0\rangle = b|0\rangle = 0$ . Using the simplified expression for  $\zeta$  in terms of  $\delta\psi$  and the background Dirac field, the two point function can be reduced to the expression at  $\vec{r}=0$

$$\langle \zeta(t, \vec{x})\zeta(t, \vec{x}) \rangle = f^2(t) \frac{\bar{\psi}\psi}{4} \langle \delta\bar{\psi}\delta\psi \rangle. \quad (28)$$

This finally provides the expression for the power-spectrum

$$\mathcal{P}(k) \sim \sum_h \frac{ma^3(t) - 2\xi\kappa n_0}{4mn_0} \frac{k^3}{4\pi^2} \bar{v}_h(t, \vec{k}) v_h(t, \vec{k}). \quad (29)$$

Using the background solution  $\psi_g$ , we obtain the equation of motion for the spinor perturbation

$$(\gamma^I e_I^\mu i\tilde{\nabla}_\mu - m - 2\xi\kappa\bar{\psi}_g\psi_g) \delta\psi = 0 \quad (30)$$

In conformal coordinates, away from the bounce, we can write the scale-factor (23) as  $a(\eta) = (\eta/\eta_0)^2$ , with  $\eta_0 = (m\kappa n_0)^{-1/2}$ , and rescale the Dirac field as  $\tilde{\psi} = a^{3/2}\psi$ , and its perturbation as  $\tilde{\delta\psi} = a^{3/2}\delta\psi$ . We then obtain

$$(\gamma^I e_I^\mu i\tilde{\nabla}_\mu - m - 2\xi\kappa\sqrt{-g}\tilde{\bar{\psi}}_g\tilde{\psi}_g) \tilde{\delta\psi} = 0, \quad (31)$$

using the background fermion density, we arrive at the equation of motion:

$$\left( i\gamma^\mu \partial_\mu - m a(\eta) - \frac{2\xi\kappa n_0}{a^2(\eta)} \right) \tilde{\delta\psi} = 0. \quad (32)$$

Following the procedures outlined in [23], we rescale densitized spinors to  $\tilde{u} = a^{3/2}u$  and  $\tilde{v} = a^{3/2}v$  in terms of their chiral and helical components

$$\tilde{u}(t, \vec{k}) = \sum_h \tilde{u}_h(t, \vec{k}) = \sum_h \begin{pmatrix} \tilde{u}_{L,h}(\vec{k}, \eta) \\ \tilde{u}_{R,h}(\vec{k}, \eta) \end{pmatrix} \xi_h, \quad (33)$$

$$\tilde{v}(t, \vec{k}) = \sum_h \tilde{v}_h(t, \vec{k}) = \sum_h \begin{pmatrix} \tilde{v}_{R,h}(\vec{k}, \eta) \\ \tilde{v}_{L,h}(\vec{k}, \eta) \end{pmatrix} \xi_h, \quad (34)$$

having introduced the helicity 2-eigenspinor, cast in terms of the unit vector  $\hat{\vec{k}}$ , which reads

$$\xi_h = \frac{1}{\sqrt{2(1-h\hat{k}_z)}} \begin{pmatrix} h(\hat{k}_x - i\hat{k}_y) \\ i\hat{k}_x - h\hat{k}_y \end{pmatrix}, \quad \hat{\vec{k}} \cdot \vec{\sigma} \xi_h = h \xi_h, \quad (35)$$

$\vec{\sigma}$  denoting the Pauli matrices. We can now solve the Dirac equation (32) in terms of

$$\begin{aligned} \tilde{f}_{\pm h} &= \frac{1}{\sqrt{2}} [\tilde{u}_{L,h}(\vec{k}, \eta) + \tilde{u}_{R,h}(\vec{k}, \eta)] \\ \tilde{g}_{\pm h} &= \frac{1}{\sqrt{2}} [\tilde{v}_{L,h}(\vec{k}, \eta) + \tilde{v}_{R,h}(\vec{k}, \eta)]. \end{aligned} \quad (36)$$

In terms of  $\tilde{f}_h$ , we rewrite equation (32) as

$$\tilde{f}_{\pm h}'' + \left[ k^2 + m^2 a^2 + i m a' + 2\xi\kappa n_0 \left( \frac{m}{a} - i \frac{a'}{a^3} \right) \right] \tilde{f}_{\pm h} = 0, \quad (37)$$

in which ' denotes derivative with respect to conformal time. An identical system of coupled equations is recovered for  $\tilde{g}_h$ . Rescaling (37) by  $\kappa$  and then taking the limits  $\eta_0^2 \ll \kappa$  and  $\kappa m^2 \ll 1$ , provided that also  $\kappa^2 m^2 \ll \eta_0^2$  is fulfilled, equation (37) reduces to

$$\tilde{f}_{\pm h}'' + \left( k^2 - \frac{\nu^2 - 1}{4\eta^2} \right) \tilde{f}_{\pm h} = 0, \quad (38)$$

in which we have defined the parameter  $\nu$  by means of

$$\nu^2 = 1 - 8\xi\kappa/\eta_0^2. \quad (39)$$

An equation identical to (38) is then found for  $\tilde{g}_{\pm h}$ ; their solutions have been extensively studied in the literature, and for non densitized components read

$$f_{\pm h}(k, \eta) = \sqrt{\frac{-\pi k \eta}{4a^3(\eta)}} Z_{|\nu|}(-k\eta), \quad (40)$$

$Z_{|\nu|}$  denoting the Bessel functions labeled by the parameter  $|\nu|$ .



Dirac field perturbations are proportional to  $f_{\pm h}(k, \eta)$  and  $g_{\pm h}(k, \eta)$ . In the contracting epoch and on sub-horizon scales, when  $-k\eta \gg 1$ , both  $f_{\pm h}(k, \eta)$  and  $g_{\pm h}(k, \eta)$  the fermionic perturbations are oscillatory and suppressed by a factor  $a^{3/2}(\eta)$ . The cosmological solution is stable, since both  $\delta\psi(\eta, \vec{x})$  and  $\delta\psi'(\eta, \vec{x})$  are not growing in time. For super-Hubble perturbations, *i.e.*  $-k\eta \ll 1$ , the solution of (38) is expressed in terms of the Bessel functions,  $Z_{|\nu|} \simeq \Gamma(|\nu|)(-k\eta)^{-|\nu|-1/2}$ , in which  $\Gamma(|\nu|)$  denotes the Euler function. For any value of  $\nu$ , this provides perturbations  $\delta\psi(\eta, \vec{x})$  and  $\delta\psi'(\eta, \vec{x})$  which decrease during the expansionary phase of the universe. The value of  $\nu$  also determines the scale-invariance of the power spectrum: far away from the bounce, the power-spectrum (29) becomes

$$\mathcal{P}(k) \simeq -\frac{k^3 |\Gamma(|\nu|)|^2}{8\pi^2 n_0} (-k\eta)^{-2|\nu|},$$

the fermion system in the contracting phase far away from the bounce yields a value of  $|\nu| = 3/2$ . This value gives the following power-spectrum

$$\mathcal{P}(k) \simeq \frac{|\Gamma(3/2)|^2}{8\pi^2 n_0 \eta^3}. \quad (41)$$

We can immediately constrain the parameter  $\xi$ , which reads

$$|\xi| = \frac{5}{32} m \kappa^2 n_0. \quad (42)$$

by imposing the COBE normalization of the CMBR power-spectrum (see also [24])  $\mathcal{P}_0 = 2,4 \times 10^{-9}$ , and then estimating  $\eta$  at present time as  $\eta_{\text{td}} = 2 H_{\text{td}}^{-1} \sqrt{a_{\text{td}}} = 4 \cdot 3^{1/3} / (H_{\text{td}}^{4/3} \eta_0^{1/3})$ , in which  $H_{\text{td}} \simeq 10^{-60} \kappa^{-1/2}$  and  $a_{\text{td}}$  are respectively the Hubble constant and the scale factor at present time. This yields for the fermion number density  $n_0 \simeq 10^{-150} m^{-3} \kappa^{-3/2}$ . Consequently, for values of the mass parameter  $m \ll 10^{-50} \kappa^{-1/2}$ , all the constraints necessary to have a scale-invariant power spectrum are satisfied. In particular, for  $m \simeq 10^{-53} \kappa^{-1/2}$  we find

$$\xi \simeq -10^{-4} \quad \rightarrow \quad \alpha \simeq -\frac{1}{\gamma} \pm \frac{\sqrt{1+\gamma^2}}{\gamma}. \quad (43)$$

Such a value of  $\xi$  and the lack of stringent constraints on the parameter  $\alpha$  and  $\gamma$  that comes from lepton-quark contact interactions, namely  $|\xi| < 10^{32}$  [20, 21], are consistent with the parameters in the non-minimal ECH theory resulting from (1) and (2). Value of the mass parameter below  $10^{-19}$  eV have been considered for dark matter candidates [25].

*Summary and conclusion* — In this letter we demonstrate that when general covariance accommodates non-minimal coupling in the fermionic sector, a contribution to torsion necessarily modifies the cosmological evolution

to yield a bounce. For the first time, we demonstrate that the very fermions that regulate the singularity, generate scale-invariant quantum fluctuations in the contracting phase. Using the arguments of Brandenberger and Finelli, we can easily match these fermionic perturbations to the scale invariant modes in the expanding phase. Our bounce is non-singular because the torsion, which is responsible for the bounce, does not receive quantum corrections. In a future paper, it will be interesting to compute the gravitational wave power spectrum, the resulting tensor to scalar ratio and its correction due to the coupling of the gravitons to the fermions.

*Acknowledgments* — We dedicate this paper to Leon Cooper, whose work continues to inspire and challenge us. SA was supported by the Department of Energy Grant de-sc0010386. This work was supported by the NSFC grant No. 11305038, the Shanghai Municipal Education Commission grant for Innovative Programs No. 14ZZ001, the Thousand Young Talents Program, and Fudan University.

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  - [26] There have been other approaches using torsion to resolve the singularity [6, 7]. However, in our work we are using non-minimal coupling to gravity induced by the topological sector and will find scale invariance
  - [27] We point out that bounce-cosmologies based on the fermion field  $\psi$  have been previously considered in the literature [12, 13], modeling coupling and self-interactions in terms of generic potential  $V(\bar{\psi}, \psi)$ . In this letter we focus more specifically on a model in which the four-fermion term originate univocally from a Dirac action non-minimally coupled to gravity endowed with a topological term.
  - [28] In companion paper [14], some of us used these findings as a starting point to discuss consequences for the fate of black hole solutions, which for a suitable choice of the some parameters of the theory may never form.
  - [29] There exists of course a *caveat*: symmetry reduction to a FLRW is allowed in presence of radiation only within the assumption of the relative smallness of the values of the electromagnetic field [22], which would otherwise spoil isotropy.
  - [30] The cosmological models whose parameters  $\alpha$  and  $\gamma$  encode a negative  $\xi$  may have application as alternative model of Inflation. Below we study the perturbation variable sourced by fermionic matter.